Examination Questions for the course "Hausdorff and Gromov-Hausdorff Distances Geometry"
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(1) Definition of topology and topological space, induced topology, subspace of topological space, discrete and anti-discrete topologies, metric spaces and metric topology, standard topology on Euclidean space.

(2) Base of topology, cover of set and subset, Zariski topology, Sorgenfrey topology, subbase of topology.

(3) Disjoint union of topological spaces, Cartesian product of topological spaces, Tychonoff or product topology, quotient topology and quotient space, Vietoris topology.

(4) Continuous mapping, homeomorphism, embedding, convergence of sequences.

(5) Closure, interior, boundary, dense subsets, separability.

(6) Separated or Hausdorff topological space, connected and disconnected topological spaces, connected components, path-connected topological spaces.

(7) Open cover, subcover, compact and sequentially compact topological spaces, bounded metric spaces.

(8) Hyperspaces.

(9) Distance, pseudometric or semimetric, metric, open ball, closed ball, sphere, distance to a nonempty subset, open and closed r-neighborhoods of a nonempty subset, diameter of a subset, natural quotient of pseudometric space.

(10) Lipschitz mappings, Lipschitz constant, dilatation of a Lipschitz mapping, uniform continuity, bi-Lipschitz mappings, isometric mapping, isometry, isometry group.

(11) Standard constructions of metrics: multiplying metric by number, adding constant to metric, induced distance, semidirect product of metric spaces, examples, Levenshtein distance.

(12) Elements of graph theory, metric construction for graphs, distance on connected graph, distance on connected weighted graph, Cayley graph of group.

(13) Quotient pseudometric and metric, generalized distance, disjoint union of generalized spaces, maximal pseudometric, the relation between quotient and maximal pseudometrics and metrics, isometries groups actions and quotient spaces, metrized graphs, polyhedron spaces.

(14) Convergence of sequences and completeness, fundamental sequences, completion of a metric space, equivalence of compactness and sequential compactness for metric spaces, completeness and total boundedness equivalent to compactness for metric spaces.

(15) Canonical isometric embeddings of metric spaces, Frechet-Kuratowski embedding to the space of bounded continuous functions, Frechet embedding of a separable metric space to the space of bounded sequences.

(16) Curves in a topological space, parameter of a curve, reparametrization, polygonal line in a metric space, its edges, length of edge, length of polygonal line, length of curve in metric space, rectifiable curves, properties of the length functional.

(17) Intrinsic metric, generalized intrinsic pseudometric, maximal and minimal generalized pseudometrics, minimum of generalized intrinsic pseudometrics, quotients of generalized intrinsic pseudometric spaces.

(18) Hopf-Rinow condition, Hopf-Rinow theorem Part 1.

(19) Convergence and uniform convergence in terms of the corresponding product spaces, limits of sequences of Lipschitz mappings, arc-length and uniform curves, reparametrizations, uniform reparametrizations, Arzela-Ascoli theorem.

(20) Shortest curves and geodesics, existences theory for shortest curves, geodesic metric space, midpoints and ε-midpoints, existence of shortest curves in term of midpoints, intrinsic metrics and ε-midpoints.

(21) Simple graphs, finite graphs, vertices, edges, isomorphism of graphs, adjacency, incidence, neighborhood of a vertex, subgraph, spanning subgraph, complete graph, subgraph generated by vertices, subgraph generated by edges.

(22) Graph theory: walk, degenerate and non-degenerated walks, open and closed walks, trail, path, circuit, cycle, connected graph, components of a graph, forest, tree, weighted graph, the weight of a subgraph, the weight of trail, the weight of walk.

(23) Operations on graphs, union, disjoint union, intersection, difference, deleting edges, deleting vertices, quotient graphs, quotient by edge, splitting vertex, splitting off vertex.

(24) Graphs in metric spaces, the length of an edge, the length of a graph, the length of minimum spanning tree, minimum spanning tree, the length of Steiner minimal tree, Steiner minimal tree, the length of minimal filling, minimal filling.

(25) mst-spectrum of finite metric space, calculation of mst-spectrum in terms of partitions.

(26) Graphs with boundaries, boundary (fixed) vertices, interior (movable) vertices, networks, parameterizing graphs of networks, boundary of a network, the length of a network, splitting and splitting off for networks, full Steiner tree, Steiner minimal trees existence in boundedly compact metric spaces.

(27) Hausdorff distance, equivalence of three definitions, triangle inequality for Hausdorff distance, Hausdorff distance is a metric on the set of all closed bounded nonempty subsets.

(28) Coincidence of Vietoris topology and metric topology generated by Hausdorff distance on the set of all compact subsets.

(29) Limits theory for nonempty subsets, definition of $\limsup$ and some its properties (equivalent definitions), definition of $\liminf$ and some its properties (equivalent definitions), convergence w.r.t. Hausdorff distance (Hausdorff convergence) and calculating $\liminf$, Hausdorff convergence of singletons.

(30) Limits theory for nonempty subsets, definition of lim, Hausdorff convergence implies existence of lim, the cases of decreasing and increasing sequences that are Hausdorff converging, equivalence of Hausdorff convergence and existence of lim in compact spaces, the cases of decreasing and increasing sequences in compact spaces.

(31) Convergence in complete metric spaces, simultaneous completeness (total boundness, compactness) of the original space and the hyperspace of all closed bounded nonempty subsets.

(32) Inheritance of the property to be geodesic for compact space and the hyperspace of all its closed nonempty subsets.

(33) Realization of a pair of metric space, Gromov–Hausdorff distance, admissible metric on disjoin union of metric spaces, calculation of Gromov–Hausdorff distance in terms of admissible metrics.

(34) Triangle inequality for Gromov–Hausdorff distance, positive definiteness of Gromov–Hausdorff distance for isometry classes of compact spaces, counterexample for boundedly compact spaces.

(35) Gromov–Hausdorff distance for separable spaces in terms of their isometric images in ℓ_∞, relations, distortion of a relation between metric spaces, correspondences, Gromov–Hausdorff distance in terms of correspondences.

(36) ε-isometries and Gromov–Hausdorff distance.

(37) Irreducible correspondences between sets, existence of irreducible correspondences, irreducible correspondences as bijections of partitions of sets, Gromov–Hausdorff distance in terms of irreducible correspondences.

(38) Gromov–Hausdorff distance between 2- or 3-point metric spaces, simple general properties of Gromov–Hausdorff distance.

(39) Gromov–Hausdorff convergence, inheritance of metric and topological properties while Gromov–Hausdorff convergence.

(40) Gromov–Hausdorff space (GH-space), distortions of a correspondence and its closure, calculating GH-distance in terms of closed correspondences, compactness of the set of all closed correspondences for compact metric spaces, continuity of distortion for compact metric spaces, optimal correspondences, existence of closed optimal correspondences for compact metric spaces, GH-space is geodesic.

(41) Cover number and packing number, there relations, total boundness of families of compact metric spaces in terms of cover and packing numbers.

(42) Isometric embedding of all compact metric spaces from a totally bounded family to the same compact subset of ℓ_∞, completeness of GH-space, separability of GH-space.

(43) mst-spectrum in terms of GH-distances to simplexes, Steiner problem in GH-space.

(44) GH-distance to simplexes with more points, GH-distance to simplexes with at most the same number of points.

(45) Generalized Borsuk problem, solution of Generalized Borsuk problem in terms of GH-distances, clique covering number and chromatic number of simple graphs, their dualities, calculating these numbers in terms of GH-distances.