March 30, 2026
Conference "Lomonosov Readings"
from 4:45 pm to 6:20 pm, room 16-10, and ZOOM-translation

Elena A. Kudryavtseva
Bifurcations of integrable systems through resonant equilibria in rigid body dynamics

The talk is devoted to bifurcations arising in integrable systems with two degrees of freedom, dependent on parameters. Examples of such systems include many integrable cases of rigid body dynamics. Suppose that an unperturbed Hamiltonian system (with a zero parameter value) has an equilibrium. This is a point at which the linear parts of the Hamilton function H and the additional first integral are zero. Then, the quadratic parts of these functions at this point generate a pair of commuting Hamiltonian linear operators. These operators generate a commutative subalgebra (with respect to the commutator operation) of the Lie algebra $\mathrm{sp}(4,\mathbb{R})$. An equilibrium is called non-degenerate if this commutative subalgebra satisfies two conditions: 1) it is two-dimensional, 2) it contains an operator with a simple spectrum. Non-degenerate equilibria are well known in any dimension: by the Eliasson–Wey theorem, they are given by a set of quadratic first integrals of the form $f_i = (p_i^2+q_i^2)/2$ (elliptic type), $f_j = p_j q_j$ (hyperbolic type), $f_{2k-1} = \mathrm{Re}((p_{2k-1}+ip_{2k})(q_{2k-1}-iq_{2k}))$, $f_{2k} = \mathrm{Im}((p_{2k-1}+ip_{2k})(q_{2k-1}-iq_{2k}))$ (focus-focus type) in some local curvilinear symplectic coordinate system.

The speaker introduced the notion of a semi-toric singularity of an integrable system. Together with L.M. Lerman, a semi-local classification of such singularities (in small neighborhoods of compact orbits of the Hamiltonian $\mathbb{R}^n$-action) was obtained (2024) for “generic” real-analytic integrable systems with $n = 2, 3$ degrees of freedom. It turned out that, in addition to non-degenerate singularities, singularities arise at which only one of the non-degeneracy conditions 1) and 2) is violated and the spectrum is resonant. Specifically, the spectrum contains either a pair of coinciding eigenvalues ??(a 1:1 or 1:-1 resonance, so condition 2) for the spectrum to be simple is violated) or a pair of commensurate eigenvalues ??(an m:n resonance, and condition 1) for the subalgebra to be two-dimensional is violated).

We compute resonances corresponding to (degenerate) equilibria of systems in rigid body dynamics. We give formulae for standard momentum maps describing many of these bifurcations. We will also describe semi-local and semi-global topological invariants for many bifurcations.

Georgy I. Sharygin
The full symmetric Toda system: symmetries and the Li-Bianci criterion

In addition to Liouville's famous criterion for the integrability of Hamiltonian systems, other methods were proposed in the 19th century to determine the integrability of a system of differential equations in quadratures. One such criterion, the Lie (or Lie-Bianchi, after the Italian geometer who popularized this result in the early 20th century) criterion, is based on the study of the Lie algebra of symmetries (i.e., vector fields that preserve the given equation) of a system: if it is possible to identify in it a finite-dimensional solvable subalgebra with dimension equal to the dimension of the space, then the system is integrable in quadratures. I will describe how this criterion works using the example of a popular dynamical system, the complete symmetric Toda system. The report is based on joint work with Yu. Chernyakov and D. Talalaev.

SCIENTIFIC SEMINAR
“DIFFERENTIAL GEOMETRY AND APPLICATIONS”

headed by Academician of RAS Anatoly T. Fomenko

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