**Monday, November 17, 2025**
**from 4:45 p.m. to 6:20 p.m. (Moscow time)**
**room 16-10 and ZOOM translation**

# Gleb V. Belozerov

## *Multidimensional integrable billiards. Their singularities and properties*

In the last few decades, the qualitative theory of integrable Hamiltonian systems (IHS) has been actively developing, studying their topological and trajectory properties. The most visual IHS are integrable billiards and their generalizations. According to recent results by V.V. Vedyushkina, I.S. Kharcheva, and A.T. Fomenko, billiard books (i.e., systems on $CW$ complexes glued from several plane confocal domains) model the Liouville foliations of many important IHS with two degrees of freedom on nonsingular surfaces of constant energy from other scientific fields.

The present report is devoted to multidimensional billiards bounded by confocal quadrics. The following dynamical system is considered: A point particle of unit mass moves within a compact domain in Euclidean $n$-dimensional space under the action of a Hooke's potential field of coefficient $k$, reflecting off the boundary of the domain absolutely elastically. Such a system is Liouville integrable in the piecewise-smooth sense. This talk will describe the Liouville foliation near non-degenerate singular fibers of this system for several of the most difficult billiard tables to study. The answer is obtained in the form of almost direct products of 2-atoms.

It turns out that in the case $k \geqslant 0$, one of the action variables of the system is well-defined in the neighborhood of a non-degenerate saddle fiber of minimum rank. This observation allowed the author and A.T. Fomenko to give a new proof of Staude's construction of a triaxial ellipsoid using a string. This construction can be generalized to the case of arbitrary dimension.

The talk also describes the topological types of non-singular isoenergy billiard surfaces in a Hooke's potential field inside an $n$-axial ellipsoid in $\mathbb{R}^n$. As it turns out, all such surfaces are homeomorphic either to the sphere $S^{2n-1}$ or to a direct product of the form $S^{i-1} \times S^{2n-i}$.

**SCIENTIFIC SEMINAR**
**"DIFFERENTIAL GEOMETRY AND APPLICATIONS"**

**headed by Academician of RAS Anatoly T. Fomenko**

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