

September 22, 2025, from 4:45 p.m. to 6:20 p.m. (Moscow time)
room 16-10 and broadcast via ZOOM

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*Delone sets: local rules,
countable families and periodicity*

In the early 1930s, B.N. Delone (Delaunay) introduced the concept of an (r, R) -system — a set of points in Euclidean space with fixed radii r and R of discreteness and coverage, respectively. These sets are a generally accepted geometric model of the atomic structure of a solid. Since the mid-1980s, (r, R) -systems have been called Delone sets.

Since the 1970s, research began on the local theory of crystal structures, motivated by the fundamental question of why, as a result of physical processes occurring during crystallization, an atomic structure with a crystallographic group is formed. Local conditions for the crystallographicity of a given Delone set were found.

In connection with the discovery of quasicrystals, research began on local rules that ensure not only crystallinity but also the quasi-periodicity of a Delone set.

The main part of the talk consists of a proof of the theorem: if a family of Delone sets with a given local rule is at most countable, then among the Delone sets in this family there is a crystal, that is, a set with a crystallographic symmetry group.

As a special case, the theorem implies L. Danzer's result: if the local rule is such that the family it defines consists of a single set (up to congruence), then this set is a crystal. Another consequence of the theorem relates to so-called aperiodic families, i.e. those that contain only non-periodic sets. An example of an aperiodic family is the family of the vertex sets in the famous Penrose tilings. As a consequence of the theorem, every aperiodic family is uncountable.

**SCIENTIFIC SEMINAR
“DIFFERENTIAL GEOMETRY AND APPLICATIONS”**

headed by Academician of RAS Anatoly T. Fomenko

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