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Argument shift method in $U\mathfrak{gl}_d$

Argument shift method is a well-known method of obtaining Poisson-commutative subalgebras in the algebra of smooth functions $C^\infty(X)$ of a Poisson manifold X. It is based on the observation that if a vector field ξ and the Poisson bivector π on X verify the equations

$$\mathcal{L}_\xi \pi \neq 0,\ \mathcal{L}_\xi^2 \pi = 0,$$

(here $\mathcal{L}_\xi$ denotes the Lie derivative), then the elements $\mathcal{L}_\xi^p(f)$ Poisson-commute with each other for all $p \geq 0$ and all smooth Casimir functions f. For instance when $X = \mathfrak{g}^*$ (the dual space of a Lie algebra) and the Poisson structure is the canonical Lie–Poisson structure on $\mathfrak{g}^*$, we can take the vector field ξ to be constant with respect to affine coordinates on $\mathfrak{g}^*$. This situation was first considered by Mischenko and Fomenko; for a generic vector field ξ this method gives a maximal Poisson-commutative subalgebra A_ξ in the symmetric algebra $S(\mathfrak{g})$; A_ξ is often called *Mischenko–Fomenko algebra*.

In 1991 E. B. Vinberg asked, whether it was possible to find a commutative subalgebra $\hat{A}_\xi$ in the universal enveloping algebra $U\mathfrak{g}$, such that the image of $\hat{A}_\xi$ in $S\mathfrak{g}$ under the canonical isomorphism of associated graded algebra of $U\mathfrak{g}$ and $S\mathfrak{g}$ would be equal toA_ξ. This question has been solved by various people, the best known construction of the *quantum Mischenko–Fomenko algebra* $\hat{A}_\xi$ being that of Rybnikov. However, finding an element in Rybnikov's algebra that will correspond to a particular $\xi^p(f)$ in A_ξ is not easy.

In my talk I will describe a method of quantising the elements $\xi^p(f)$ (i.e. raising it to $\hat{A}_\xi \subset U\mathfrak{g}$ for $\mathfrak{g} = \mathfrak{gl}_d$. It is based on a systematic use of the *quasiderivation operation* of Gurevich, Pyatov and Saponov $\hat{\xi}$ on $U\mathfrak{gl}_d$ in the stead of usual directional derivative ξ (we assume that the coefficients of $\hat{\xi}$ coincide with those of ξ). Namely, we can prove the following result

Theorem. *Let $\hat{f} \in U\mathfrak{gl}_d$ be a central element, $p \geq 0$; then the element $\hat{\xi}^p(\hat{f})$ is in the quantum Mischenko-Fomenko algebra A_ξ. In particular, all such elements commute with each other.*

The talk is based on a joint work with Yasushi Ikeda, `arXiv:2307.15952`.

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