September 25, 2023

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Geometry of the Lie group in the group analysis of the one-dimensional kinetic equation

A phenomenon under consideration discovered in the framework of a study on the problem of transition from kinetic equations to continuum medium equations. As part of this study, it was necessary to deal with the geometric interpretation of the obtained classification of one-dimensional kinetic equations.

It turned out that one can define a geometry on a Lie group and discuss its geometric properties, fundamentally different from what E. Cartan once discussed. Namely, on a Lie group of dimension n there are $n(n+1)/2$-dimensional linear space of metrics that are invariant under the group. All they are quadratic forms (with constant coefficients) of a set of n differential forms of the first order, also invariant under the group.

If we consider the matrix of coefficients of these differential forms, then the inverse matrix generates Lie algebra on this group (which is called dual), distinct from the generating algebra and defined by the commutation condition for any two operators of these two algebras.

Lines on a group that are invariant with respect to this group are trajectories of one-parameter subgroups generated by the dual algebra, and these lines turn out to be spirals (that is, all their curvatures are constant) in any of the metrics that are invariant under the group under consideration. Wherein the Frenet equations of these curves are reduced to a linear expression of the next curvatures in terms of the previous ones with a constant matrix defined in terms of the structure constants of the dual algebra. The same reduction can be done with the Riemann and Ricci tensors.

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