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Nonassociative algebras of minimal cones

It was observed by Wu-yi Hsiang in [2] that any cubic minimal cone, i.e. a minimal submanifold of $\mathbb{R}^n$ given as the zero-level set of a homogeneous polynomial $u(x)$ of degree 3, is a solution of certain nonlinear PDE. But by Hsiang's remarks, "...the algebraic difficulties involved in such a problem are rather formidable". In particular, a direct approach works hardly even for the trivial dimension $n = 2$. Using an elegant trick based on an invariant theory, Hsiang was able to construct four explicit solutions in dimensions $n = 5$, 8, 9, 15, and he also and proposed to find a unified algebraic approach for the general case.

In my talk, I will discuss how a classification of cubic minimal cones and the corresponding PDE problem can be appropriately translated into a pure nonassociative algebra context. The corresponding nonassociative (Hsiang) algebras satisfy the defining identity $x^2x^2 + 4xx^3 - 4\langle x, x\rangle x^2 = -\frac{4}{3}\lambda\langle x^2, x\rangle x$, where $\lambda \in \mathbb{R}$ and the bilinear form $\langle\cdot,\cdot\rangle$ satisfies a Killing type identity: $\langle xy, z\rangle = \langle x, yz\rangle$. Despite considerable recent progress, the full classification of Hsiang algebras remains open. I will explain some of my recent results, in particular, connection between Hsiang algebras and Jordan algebras, Clifford algebra representation theory [5], [7], and axial algebras [1], [3]. I will also mention some unexpected connections of Hsiang algebras with a completely different context of viscosity solutions of fully nonlinear PDE [4], [6].

References

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