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Algebraic Morse functions and the realizability of any arrangement of ovals on the plane as an algebraic curve

The talk is devoted to a problem related to Hilbert's 16th oval problem. We show that any arrangement of ovals in the plane can be realized (up to isotopy) as an algebraic curve of degree $2k$, where k is the number of ovals. Moreover, there exists a realizing polynomial of the form $|P|^2 - |Q|^2$, where P and Q are coprime polynomials (of degrees k and less, respectively) in one variable $z = x + iy$ with complex coefficients, and the number of roots of the polynomial PQ is equal to k. Moreover, the degree $2k$ of the curve is the best for realizing polynomials of the indicated form, i.e., for any arrangement of ovals, it cannot be reduced while preserving the form $|P|^2 - |Q|^2$ of the realizing polynomial.

Moreover, any Morse function F on a two-sphere that realizes a given arrangement of k ovals as its set of zeros and has the minimum number of critical points (equal to $2k$) is fiberwise equivalent to some function of the form $|P/Q|$. Moreover, the space of all such Morse functions F is homotopy equivalent to the space of functions of the form $|P/Q|$.

SCIENTIFIC SEMINAR
"DIFFERENTIAL GEOMETRY AND APPLICATIONS"

headed by Academician of RAS Anatoly T. Fomenko

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