Секция «Геометрия и топология»

Почти-вложения графа K_5 без ребра в плоскость.

Научный руководитель – Миллионщиков Дмитрий Владимирович

Гараев Тимур Рустемович
Студент (специалист)
Московский государственный университет имени М.В.Ломоносова,
Механико-математический факультет, Москва, Россия
E-mail: turgar04@yandex.ru

Denote $S^2 := \{(x, y, z) : \max\{|x|, |y|, |z|\} = 1\}$.

A continuous map $g : K \to S^2$ of a graph K is called an **almost embedding** if $g(\alpha)\cap g(\beta) = \varnothing$ for any two disjoint edges $\alpha, \beta \subseteq K$.

Denote $[n] := \{1, 2, \dots, n\}$.

Denote by K_5 the complete graph with the vertices set [5]. Denote by (l, k) the edge between vertices l and k in a graph. Denote by $K_5\backslash(4, 5)$ the graph obtained from the graph K_5 by deleting the edge $(4, 5)$.

Denote by Σ^1 the complete graph K_3 with the vertices set [3] ordered by cyclic permutation (1 2 3).

Let us define the **linking number** $\text{lk}(\gamma^1, \gamma^0) \in \mathbb{Z}$ of disjoint oriented closed polygonal line γ^1 on S^2 and ordered pair γ^0 of points in $S^2\backslash\gamma^1$. Take an oriented polygonal line L in general position with γ^1 such that ∂L with order inherited from L coincides with γ^0. By $\text{lk}(\gamma^1, \gamma^0)$ denote the sum of the signs of the intersection points of L and γ^1. It is well known that the linking number is well defined.

Theorem 1. *([4], Theorem 1]). For any almost embedding $g : K_5\backslash(4, 5) \to S^2$ the absolute value of $lk(g|_{\Sigma^1}, (g(4), g(5)))$ does not exceed* 3.

It is well-known that for any almost embedding $g : K_5\backslash(4, 5) \to S^2$ the absolute value of linking coefficient $\text{lk}(g|_{\Sigma^1}, (g(4), g(5)))$ is odd. There are similar results for embeddings of graphs into $\mathbb{R}^3$, see survey [1]. Theorem 1 gives a constraint on the absolute value of the linking coefficient. Theorem 1 disproves conjecture 1.6(a) in the first arXiv version of [2]. We conjecture that for any almost embedding $g : K_5\backslash(4, 5) \to S^2$ the absolute value does not exceed 1.

Some theorems in topology of the plane have technical proofs, e.g. Jordan curve theorem and completeness of the van Kampen planarity obstruction in [3]. This work is no exception.

On the report I will show the main idea of the proof of Theorem 1.

Список литературы

[1] *Skopenkov A.* Realizability of hypergraphs and Ramsey link theory // arXiv:1402.0658.

[2] *Karasev R., Skopenkov A.* Some ‘converses’ to intrinsic linking theorems // arXiv:2008.02523.

[3] *Michael J. Pelsmajer, Marcus Schaefer, and Daniel ˇStefankoviˇc.* Removing even crossings. J. Combin. Theory Ser. B, 97(4):489–500, 2007.

[4] *Garaev T.* On drawing K_5 minus an edge in the plane // arXiv:2303.14503.