December 12, 2022

Vasiliy O. Manturov

(joint work with I.M.Nikonov)

Braids and knots. Maps from the cylinder to virtual category

In virtual knots, many new invariants were discovered by V.O.Manturov over the last decades; these invariants are principally new in low-dimensional topology. Among them, one can mention the parity bracket, the invariant valued in linear combinations of knot diagrams. There are diagrams of complicated virtual knots K for which $[K] = K$, in other words, the value of the bracket on the diagram K is equal to K with coefficient 1. This allows one to reduce various problems about knots to questions about a single diagram of a knot. For example, if K' is equivalent to K (generates the same knot) then the equality $[K'] = K$ holds; by construction this means that K "lies inside" K'. This situation is similar to the words in free groups: the unique reduced representative of a given word class can be obtained from any equivalent word by canceling letters. Such (bracket-like) invariants appear in virtual knot theory because of the parity phenomenon (or existence of homology group of the ambient space); they formally do not work in the case of classical knot theory (the plane where diagrams are drawn has no non-trivial homology); applying parity theory to knots in cylinder does not lead to striking invariants. Besides the brackets, the parity allows one to enhance various known invariants of classical knots and to construct "non-commutative invariants" and concordance invariants (sliceness obstructions) in an easy way.

The aim of the present talk is to construct "functorial maps" from knots in the full torus $S^1 \times D^2$ and knots in the thickened torus $T^2 \times D^2$ to analogues of virtual knots of high genus. This allows one to apply the virtual knot theory techniques to classical objects — links with one trivial components.

The talk consists of two parts. In the first part we work with classical braids and "exchange" one strand or two strands for increasing the genus of the ambient space (first we get cylindrical braids and then increase genus even higher and construct new braid presentations, in particular, we strengthen the Burau representation). In the second part we deal with links in the full torus and in the thickened torus (which can be treated as links with one or two trivial components in 3-space) and discuss various examples of applying virtual knot invariants in the classical case. After that we address several problems and ongoing research projects, among them:

(1) application of the above methods to classical 1-component knots;

(2) cobordisms and concordance;

(3) Legendrians knots;

(4) Θ-graphs embedded in 3-space and their invariants.

SCIENTIFIC SEMINAR
"DIFFERENTIAL GEOMETRY AND APPLICATIONS"

headed by Academician of RAS Anatoly T. Fomenko

The seminar takes place online in ZOOM on Mondays
from 4:45 p.m. to 6:20 p.m. (Moscow time)
The zoom-ref is provided only to registered persons
To be registered, ask any participant of our seminar to endorse you
Announcements of previous talks can be found on the seminar website
`http://dfgm.math.msu.su/chairsem.php`