November 21, 2022

Gleb V. Belozerov

Geodesic flow at the intersection of non-degenerate confocal quadrics

According to the Jacobi–Schall theorem, the tangent lines drawn at all points of a geodesic curve on a quadric in n-dimensional Euclidean space are tangent, as well as to the given quadric, to $n-2$ other confocal quadrics, which are the same for all points of the geodesic curve. This theorem implies the integrability of a geodesic flow on an ellipsoid.

V. Kibkalo researches the integrability of a geodesic flow at the intersection of several confocal quadrics. He proved that the geodesic flow at the intersection of $n-2$ confocal quadrics is a completely integrable Hamiltonian system.

It turns out that the result remains true if we consider the geodesic flow at the intersection of an arbitrary number of non-degenerate confocal quadrics.

Theorem 1 (Belozerov). Let $Q_1, \dots, Q_k$ be nondegenerate confocal quadrics of different types in $\mathbb{R}^n$ and $Q = \bigcap\limits_{i=1}^{k} Q_i$, then

(1) the geodesic flow on Q is quadratically integrable;

(2) the tangent lines drawn to all points of the geodesic curve on Q are tangent, as well as to the quadric $Q_1, \dots, Q_k$, to other $n-k-1$ confocal quadrics with $Q_1, \dots, Q_k$, which are the same for all points of the geodesic curve.

Remark. Geodesic curves at the intersection of nondegenerate confocal quadrics are, generally speaking, not geodesic curves on any of the quadrics $Q_1, \dots, Q_k$. Therefore, Theorem 1 is not a consequence of the classical Jacobi–Schall theorem.

According to Theorem 1 and the result of V. Kozlov about integrable geodesic flows on two-dimensional surfaces, the connected component of the compact intersection of $n-2$ confocal quadrics is homeomorphic either to a torus $\mathbb{T}^2$ or to a sphere S^2. And both of these cases are realized. Note that this result was obtained earlier by V. Kibkalo. However, it is possible to describe the homeomorphism class of any compact intersection of non-degenerate confocal quadrics. It turns out that any such intersection is homeomorphic to a direct product of spheres.

Similarly to Theorem 1, I describe the class of potentials V in $\mathbb{R}^n$ such that the restriction of the system with the potential to any intersection of confocal quadrics would remain completely integrable. In particular, the Hooke potential belongs to this class.

SCIENTIFIC SEMINAR
"DIFFERENTIAL GEOMETRY AND APPLICATIONS"

headed by Academician of RAS Anatoly T. Fomenko

The seminar takes place online in ZOOM on Mondays
from 4:45 p.m. to 6:20 p.m. (Moscow time)
The zoom-ref is provided only to registered persons
To be registered, ask any participant of our seminar to endorse you
Announcements of previous talks can be found on the seminar website
`http://dfgm.math.msu.su/chairsem.php`