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Three-dimensional billiards inside domains bounded by confocal quadrics

The report will consider two types of three-dimensional billiards bounded by confocal quadrics: an ordinary billiard (no forces act on a material point) and a billiard in the Hooke potential field. Both of these systems are integrable Hamiltonian systems in the piecewise smooth sense.

In the absence of external forces on the set of billiards bounded by confocal quadrics, a weak equivalence relation was introduced and a classification theorem was proved. A classification of billiard "tables" is obtained with respect to their combinatorial equivalence (that is, as cellular complexes). For each of the discovered "tables", the homeomorphism class of the corresponding nonsingular isoenergetic five-dimensional manifold is found. It turned out that this is either a 5-dimensional sphere, or a direct product of a circle and a 4-sphere, or a direct product of a 2-sphere and a 3-sphere. In addition, it was possible to describe the singularities (bifurcation atoms) of three-dimensional Liouville tori that arise in the phase space of three-dimensional billiards.

A billiard in the Hooke potential field will be considered inside a table bounded by an ellipsoid and two segments of a two-sheeted hyperboloid. A bifurcation diagram has been constructed for this system, and pre-images of points of the moment map have been described. As it turned out, this billiard is closely related to the flat billiard in the the Hooke potential field inside the ellipse. This connection helped to describe the topology of the Liouville foliation near singular points.

SCIENTIFIC SEMINAR
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