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*Cloud geometry in the Gromov–Hausdorff class*

We discuss some geometric results of studying a proper class of all metric spaces endowed with the Gromov–Hausdorff distance. We will also consider a number of quotient spaces of this class. We begin with a brief summary of the theory that makes it possible to endow certain proper classes (filtered by sets) with an analogue of topology. Note that the Gromov–Hausdorff distance is a generalized pseudometric, i.e. the distance between different spaces can be equal to zero, and also be infinite. For such distances, it is natural to partition the entire space into subclasses consisting of spaces at finite distances from each other. We call such subclasses clouds. The famous work by M.Gromov “Metric structures for Riemannian and non-Riemannian spaces” states that clouds are complete and contractible. We were able to easily prove the first statement, but the second, in the case of clouds consisting of unbounded spaces, is doubtful. The thing is that the natural “contraction”, based on the similarity transformation, which multiplies all the distances of the considered spaces by a given positive number, behaves quite exotically. As we have established, with such a transformation, space can jump out of its cloud. Moreover, even in the case of invariant clouds, the continuity of the family of similarity maps is not obvious and, possibly, does not hold. We present a number of results from the study of stationary subgroups of the multiplicative group of positive real numbers, acting by the corresponding similarities. In addition, we will discuss a number of other geometric results, such as the connectedness of certain spheres, the existence of generic spaces of any cardinality, and the possibility of isometrically embedding any metric space in the Gromov–Hausdorff class.

SCIENTIFIC SEMINAR

“DIFFERENTIAL GEOMETRY AND APPLICATIONS”

headed by Academician of RAS Anatoly T. Fomenko

The seminar takes place online in ZOOM on Mondays  
from 4:45 p.m. to 6:20 p.m. (Moscow time)

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