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Global topological invariants of integrable Hamiltonian systems

We study the topology of Lagrangian fibrations with singularities arising in integrable Hamiltonian systems (IHSs). Two IHSs are said to be topologically equivalent if the corresponding Lagrangian fibrations with singularities are homeomorphic (all fibers are assumed to be compact). As examples of topological invariants of IHSs, we consider the base of the fibration with natural stratification (the so-called bifurcation complex introduced by A.T. Fomenko in 1988), as well as the topological types of singularities corresponding to the strata of this complex. The question arises: what additional topological invariants must be added, in order to obtain a complete topological invariant of the IHSs? This question can be reformulated as follows. Suppose that two IHSs are roughly equivalent in the sense of A.T. Fomenko, i.e., there exists a homeomorphism between the bifurcation complexes (fibration bases), which lifts locally (i.e., in a small neighborhood of any point of the base) to a fiberwise homeomorphism. When does a given homeomorphism between the bases lifts to a (global) fiberwise homeomorphism?

The solution was known only in some special cases, e.g. for toric IHSs (in terms of the Delzant polytope), and for almost-toric IHSs with 2 degrees of freedom. In all these cases, the absence of hyperbolic singularities was assumed. In general case, N.T. Zung solved (in 2003) a similar problem, where the IHSs are assumed to be not only roughly equivalent in the sense of A.T. Fomenko, but also roughly equivalent in the sense of N.T. Zung. The latter means the existence of a homeomorphism h between the bases, a collection of open subsets U_i of the base, and a set of lifts Φ_i of the homeomorphism h over U_i such that each pair of lifts Φ_i and Φ_j are isotopic on the intersection $U_i \cap U_j$ in the space of lifts. However, the question of when a rough equivalence in the sense of A.T. Fomenko implies a rough equivalence in the sense of N.T. Zung remained open.

In the talk, we will formulate open problems, describe known results and our solution of the problem for the IHSs with 2 degrees of freedom with non-degenerate singularities of any type (saddle, center, focus-focus, center-center and saddle-center) except for the saddle-saddle, provided that the bifurcation complex is simply-connected. We will show that such an IHS has a periodic first integral, and the (additional) topological invariant is a set of Delzant polygons and an integer (equal to the label n of the Fomenko–Zieschang invariant for the restriction of the foliation to a level set of the periodic integral).

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