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Joint work with

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Uniform Semiclassical Plancherel–Rotach type asymptotics of multiple orthogonal Hermite polynomials

The Hermitian type orthogonal polynomials $H_{n_1,n_2}(z,a)$ are determined by the pair of recurrence relations for the polynomials $H_{n_1+1,n_2}(z,a)$, $H_{n_1,n_2+1}(z,a)$, $H_{n_1,n_2-1}(z,a)$, $H_{n_1,n_2-1}(z,a)$, $H_{n_1,n_2}(z,a)$. We obtain a uniform asymptotics of diagonal polynomials $H_{n,n}(z,a)$ in the form of an Airy function for $n \gg 1$, which is a far-reaching generalization of the Plancherel–Rotach asymptotic formulas for standard Hermitian polynomials. We discuss one of the possible approach which we call "real-valued semiclassics for asymptotics with complex-valued phases", (another approach based on the construction of decompositions of bases of homogeneous difference equations was recently developed by A.I.Aptekarev and D.N.Tulyakov). This approach can be applied to construct asymptotic formulas for various orthogonal polynomials. Introducing an artificial small parameter $h = \mathrm{O}(1/n)$ and a continuous function $\varphi(x,z,a)$, such that $H(z,a) = \varphi(kh,z,a)$. We reduce the described to a pseudo-differential equation for φ, where x is a variable and (z,a) are parameters. Seeking its solution in the WKB-form, one obtains the Hamilton–Jacobi equations with *complex* Hamiltonians connected with a third-order algebraic curve. This circumstance is the main difficulty of solving the problem and, as a rule, leads to the transition from the real variable x to the complex one. In this problem, we propose a different approach based on a reduction of the original problem to three equations, two of which have asymptotics with a purely imaginary phase, and the symbol of the third one is pure real and has the form $\cos p + V_0(x) + h\,V_1(x) + \mathrm{O}(h^2)$. This ultimately allows us to represent the desired asymptotic uniformly through the Airy function of the complex but real-valued argument.

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