On Non-local Modified Gravity

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SEMINAR OF CHAIR OF DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS

Leader A. T. Fomenko

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- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold M with metric $(g_{\mu\nu})$ of signature $(1,3)$.
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
  - sphere S^3 (of constant positive sectional curvature),
  - flat space $\mathbb{R}^3$ (of curvature equal 0),
  - hyperbolic space H^3 (of constant negative sectional curvature)
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where $a(t)$ is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

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- GTR is based on Einstein-Hilbert action:

$$S = \int \left(\frac{R - 2\Lambda}{16\,\pi\,G\,c^4} + \mathcal{L}_m \right) \sqrt{-g}\; d^4x$$

where R is scalar curvature, $g = \det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and $\mathcal{L}_m$ is Lagrangian of matter.

- The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1 \tag{2}$$

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

- The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \mathrm{diag}(-\rho\, g_{00}, g_{11} p, g_{22} p, g_{33} p), \tag{3}$$

where ρ is energy density and p is pressure.

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- Using the conservation law we get

$$0 = \nabla_\mu T_0^\mu = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p). \tag{4}$$

- Since in the cosmology holds $p = w\rho$, where w is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
- The basic types of matter in the Universe are:
  - cosmic dust - $w = 0$, and $\rho_m = Ca^{-3}$.
  - radiation – - $w = 1/3$, and $\rho_r = Ca^{-4}$.
  - In this moment the ratio $\frac{\rho_m}{\rho_r} \approx 10^6$.
- From the expression for FRW metric it follows

$$R(t) = \frac{6\left(a(t)\ddot{a}(t) + \dot{a}(t)^2 + k\right)}{a(t)^2}$$

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$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}.$$

- Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a}. \tag{5}$$

- Despite to the great success of GRT in describing:
  - the precession of Merkur perihelion,
  - the bending of light in gravitational fields,
  - the gravitational redshift of light,
  - the gravitational lensing,
  - and other ...

GTR has certain deficiencies.

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- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

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There are two natural approaches:

- Dark matter and energy
- Modification of Einstein theory of gravity.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
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- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity

- Einstein General Theory of Relativity
  From action

$$S = \int \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}_m \right) \sqrt{-g} d^4 x$$

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- Under nonlocal modification of gravity we understand replacement of the scalar curvature R in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is d'Alembert operator and ∇_μ denotes the covariant derivative
- Let M be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_M \Big(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R)\,\mathcal{F}(\Box)\,\mathcal{G}(R) \Big) \sqrt{-g}\, d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of $\Box$, and Λ is cosmological constant.

- In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}\,, \quad \Box R = -\ddot{R} - 3\,H\,\dot{R}, \quad H = \frac{\dot{a}}{a}\,.$$

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Lemma 1. *For any two scalar functions $\mathcal{G}$ and $\mathcal{H}$ hold*

$$\int_M \mathcal{H}\delta(\sqrt{-g})\,\mathrm{d}^4x = -\frac{1}{2}\int_M g_{\mu\nu}\mathcal{H}\delta g^{\mu\nu}\sqrt{-g}\,\mathrm{d}^4x,$$

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$$\delta S_0 = \int_M G_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\, d^4x + \Lambda\int_M g_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\, d^4x, \tag{6}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

- Using previous theorem we find the variation of S_1,

$$\delta S_1 = -\frac{1}{2}\int_M g_{\mu\nu}\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R)\delta g^{\mu\nu}\sqrt{-g}\, d^4x$$

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Theorem 2 (EOM) *The equations of motion for system given by S are:*

$$\tilde{G}_{\mu\nu} = 0, \qquad (8)$$

where

$$\tilde{G}_{\mu\nu} = \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2}\Omega_{\mu\nu},$$

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- If we suppose that the manifold M is endowed with FRW metric, then we have just two linearly independent equations (trace and 00-equation):

$$-2\mathcal{H}\mathcal{F}(\Box)\mathcal{G} + RW + 3\Box W + \frac{1}{2}\Omega = \frac{R-4\Lambda}{16\pi G}, \quad \Omega = g^{\mu\nu}\Omega_{\mu\nu},$$

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- $\mathcal{H}(R) = R^p$, $\mathcal{G}(R) = R^q$,
- $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$,
- $\mathcal{H}(R) = R^p$, $\mathcal{G}(R) = R^q$ and $R = \mathrm{const.}$

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- It is clear from the complexity of EOM that constructing a general solution is a very ambitious hope, and we use the following ansatz,

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Theorem 3

The scaling factor of the form $a(t) = a_0\,(\sigma\, e^{\lambda t} + \tau\, e^{-\lambda t})$ is a solution of EOM in the following three cases:

Case 1: $\mathcal{F}\left(2\lambda^2\right) = 0, \quad \mathcal{F}'\left(2\lambda^2\right) = 0, \quad f_0 = -\frac{1}{128\pi G C \Lambda}$

Case 2: $3k = 4\, a_0^2 \Lambda \sigma \tau$

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Conclusion

- We consider nonlocal model of gravity with the cosmological constant Λ and without matter.
- Using the ansatz $\Box R = r R + s$ we found three types of nonsingular bouncing solutions for cosmological scale factor in the form $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
- Solutions exist for all three values of $k = 0, \pm 1$.
- Obtained solutions extend the known cases in the literature: in the first case $a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{3}} t\right)$, in the second and third case for $k = 0$ we obtain de Sitter solutions.
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where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$. For $f_0 = -\frac{\Lambda}{8\pi G}$, it is evident that f_0 plays role of the cosmological constant.

- Nonlocal term $R^{-1}\mathcal{F}(\Box)R$ is invariant under the transformation $R \to cR$, $c \in \mathbb{R}^*$.
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Theorem 4

For $k = 0$, $\alpha \neq 0$, $\alpha \neq \frac{1}{2}$ and $\frac{3\alpha - 1}{2} \in \mathbb{N}$, the scale factor $a(t) = a_0|t - t_0|^\alpha$ is a solution EOM if

$$f_0 = 0, \ f_1 = -\frac{3\alpha(2\alpha - 1)}{32\pi G(3\alpha - 2)},$$

$$f_n = 0 \quad \text{for} \quad 2 \leq n \leq \frac{3\alpha - 1}{2},$$

$$f_n \in \mathbb{R} \quad \text{for} \quad n > \frac{3\alpha - 1}{2}.$$

Theorem 5

For $k \neq 0$ the scale factor $a(t) = a_0|t - t_0|$ is a solution of EOM if

$$f_0 = 0, \quad f_1 = \frac{-s}{64\pi G}, \quad f_n \in \mathbb{R}, \quad n \geq 2,$$

where $s = 6(1 + \frac{k}{a_0^2})$.

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For $k = 0$, $\alpha \neq 0$, $\alpha \neq \frac{1}{2}$ *and* $\frac{3\alpha - 1}{2} \in \mathbb{N}$, *the scale factor* $a(t) = a_0|t - t_0|^\alpha$ *is a solution EOM if*

$$f_0 = 0, \; f_1 = -\frac{3\alpha(2\alpha - 1)}{32\pi G(3\alpha - 2)},$$

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Conclusion.

- Nonlocality $R^{-1}\mathcal{F}(\Box)R$, is invariant under the transformation $R \longrightarrow cR, \ c \in \mathbb{R}^*$.
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$$R(t) = 6\left(\alpha(2\alpha - 1)(t - t_0)^{-2} + \frac{k}{a_0^2}(t - t_0)^{-2\alpha}\right)$$

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- We consider the model given by

$$S = \int_M \Big(\frac{R - 2\Lambda}{16\pi G} + R^p \mathcal{F}(\Box) R^q \Big) \sqrt{-g}\, d^4x,$$

- with the scale factor $a(t) = a_0 e^{-\frac{\gamma}{12} t^2}, \ \gamma \in \mathbb{R}$, and consequenly we have

$$H(t) = -\frac{1}{6}\gamma\, t, \qquad R(t) = \frac{1}{3}\gamma\,(\gamma\, t^2 - 3), \qquad R_{00} = \frac{1}{4}(\gamma - R).$$

- We obtain the following relation

$$\Box\, R^p = p\,\gamma R^p - \frac{p}{3}(4p-5)\,\gamma^2\, R^{p-1} - \frac{4}{3}\,p\,(p-1)\,\gamma^3\, R^{p-2}.$$

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Theorem 6

For any $p, q \in \mathbb{N}$ trace and 00 equation are equivalent.

The trace equation is of polynomial type of degree $p + q$ in R, with coefficients depending on $f_0 = \mathcal{F}(0), \mathcal{F}(\gamma), \ldots, \mathcal{F}(p\gamma), \mathcal{F}'(\gamma), \ldots, \mathcal{F}'(q\gamma)$.

Theorem 7

(1) For $p = q = 1$, trace equation is satisfied iff $\gamma = -12\Lambda$, $\mathcal{F}'(\gamma) = 0$ and $f_0 = \frac{3}{32\gamma\pi G} - 8\mathcal{F}(\gamma)$. In this case system has infinitely many solutions.

(i2) For $(p, q) \neq (1, 1)$ system has unique solution for any γ.

(i3) The exact solutions, for $1 \leq q \leq p \leq 4$, are found.

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Theorem 9

Let in the model $\mathcal{H}(R) = R^p, \mathcal{G}(R) = R^q$, $R = \mathrm{const}$, then the solutions of EOM are given by

(i1) For $R_0 > 0$, $a(t) = \sqrt{\frac{6k}{R_0} + \sigma e^{\sqrt{\frac{R_0}{3}}t} + \tau e^{-\sqrt{\frac{R_0}{3}}t}}$,

(i2) For $R_0 = 0$, $a(t) = \sqrt{-kt^2 + \sigma t + \tau}$,

(i3) For $R_0 < 0$, $a(t) = \sqrt{\frac{6k}{R_0} + \sigma \cos\sqrt{\frac{-R_0}{3}}t + \tau \sin\sqrt{\frac{-R_0}{3}}t}$,

if $R_0 + 4 R_0 0 = 0$ and parameters σ, τ satisfy

$$\begin{array}{lll} (1) & R_0 > 0, & 9k^2 = R_0^2 \sigma\tau, \\ (2) & R_0 = 0, & \sigma^2 + 4k\tau = 0, \\ (3) & R_0 < 0, & 36k^2 = R_0^2(\sigma^2 + \tau^2), \end{array}$$

or if $\mathcal{G}(R_0)\mathcal{H}(R_0) - (R_0 - 2\Lambda)\frac{\partial}{\partial R}(\mathcal{G}(R)\mathcal{H}(R))|_{R=R0} = 0$.

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Theorem 10

Under assumptions of the previous and condition $R_0 + 4R_{00} = 0$, we have

(i1) *If $R_0 > 0$, then solutions are*

- *for* $k = 0 : a(t) \sim \exp(\lambda t)$ *(constant Hubble parameter)*
- *for* $k = +1 : a(t) = \sqrt{\frac{12}{R_0}} \cosh \frac{1}{2}\left(\sqrt{\frac{R_0}{3}} t + \varphi\right)$
- *for* $k = -1 : \ a(t) = \sqrt{\frac{12}{R_0}} \left|\sinh \frac{1}{2}\left(\sqrt{\frac{R_0}{3}} t + \varphi\right)\right|$*, where* φ *is chosen such that* $\sigma + \tau = \frac{6}{R_0} \cosh \varphi$ *and* $\sigma - \tau = \frac{6}{R_0} \sinh \varphi$.

(i2) *If $R_0 = 0$, then solutions are*

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Conclusion.

- If $R = R_0 > 0$, then there exist nonsingular solutions for all three values of $k = 0, \pm 1$, which are bounced solutions for $k = 0, 1$.
- If $R = R_0 = 0$ then there exists Milne solution $a(t) = |t + \frac{\sigma}{2}|$.
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- If $R = R_0 > 0$, then there exist nonsingular solutions for all three values of $k = 0, \pm 1$, which are bounced solutions for $k = 0, 1$.
- If $R = R_0 = 0$ then there exists Milne solution $a(t) = |t + \frac{\sigma}{2}|$.
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- Recently, we have considered classes of nonlocal gravity models with cosmological constant Λ and without matter, given by

$$\text{(M4)} \qquad S = \frac{1}{16\pi G}\int_M \big(R - 2\Lambda + (R - 4\Lambda)\,\mathcal{F}(\Box)(R - 4\Lambda)\big)\sqrt{-g}\,\mathrm{d}^4x,$$

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- We shall use some cosmological parameters from Planck 2018 results to test validity of obtained solutions for the current state of the universe. The current Planck results for the ΛCDM universe are:

- $H_0 = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter,
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where $\bar{\rho}$ and $\bar{p}$ are analogs of the energy density and pressure of the dark side of the universe, respectively.

- From the corresponding equation of state, $\bar{p}(t) = \bar{w}(t)\,\bar{\rho}(t)$, it follows

$$\bar{w} = \frac{t^{-2} - 3\Lambda^2 t^2 - 6\Lambda}{3t^{-2} + 3\Lambda^2 t^2 + 2\Lambda} \to \begin{cases} -1, & t \to \infty, \\ \frac{1}{3}, & t \to 0. \end{cases}$$

- The expressions (18) implies that $\bar{w}(t) \to -1$ when $t \to \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model,
- and $\bar{w}(t) \to 1/3$ when $t \to 0$, what corresponds to early times as for the case of radiation.
- From expression for Hubble parameter, (11), follows:
- the first term ($\frac{1}{2t}$) is the same as for the radiation dominance in Einstein's gravity, while the second term ($\frac{\Lambda t}{2}$) can be related to the dark energy generated by cosmological constant Λ.

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- At the present cosmic time $t_0 = 13.801 \cdot 10^9$ yr and $\Lambda = 0.98 \cdot 10^{-35}$ s^{-2}, both terms in (11) are of the same order of magnitude.
- Since, the value for the Hubble parameter, and $H(t_0) = 100.2$ km/s/Mpc, is larger than current Planck mission result $H_0 = 67.40 \pm 0.50 km/s/Mpc$, this cosmological solution may be of interest for the early universe with radiation dominance and for far-future accelerated expansion.

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- 2. Cosmological solution for $a(t) = A\, e^{\Lambda t^2}$, $k = 0$
  - For this solution we have

$$\dot{a}(t) = a(t)\, 2\Lambda t, \qquad \ddot{a}(t) = a(t)\, 2\Lambda(2\Lambda t^2 + 1) \tag{18}$$

  - and scalar curvature becomes

$$R(t) = 12\Lambda(4\Lambda t^2 + 1). \tag{19}$$

  - The Hubble parameter

$$H(t) = 2\Lambda\, t. \tag{20}$$

  - There is useful equality

$$\Box(R - 4\Lambda) = -12\Lambda(R - 4\Lambda), \tag{21}$$

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- R_{00} and G_{00} are:

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$$\bar{\rho}(t) = \frac{\Lambda(12\Lambda t^2 - 1)}{8\pi G}, \quad \bar{p}(t) = -\frac{3\Lambda(4\Lambda t^2 + 1)}{8\pi G}. \tag{26}$$

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- The solutions $a_1(t) = A\sqrt{t}e^{\frac{\Lambda}{4}t^2}$ and $a_2(2t) = Ae^{\Lambda t^2}$ are not contained in Einstein's gravity with cosmological constant Λ. The solution $a_1(t)$ mimics interference between expansion with radiation $a_1(t)$ and a dark energy $a_2(t)$.
- The solution $a_2(t)$ is a nonsingular bounce one and an even function of cosmic time. An exact cosmological solution of the type $a(t) = Ae^{\alpha\Lambda t^2}$, where $\alpha \in \mathbb{R}$, appears also at least in the following two models: (1) $P(R) = Q(R) = R$, and (2) $P(R) = Q(R) = \sqrt{R - 2\Lambda}$.
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- According to our solutions $a(t) = A\sqrt{t}e^{\frac{\Lambda}{4}t^2}$ and $a(t) = At^{\frac{2}{3}}e^{\frac{\Lambda}{14}t^2}$, it follows that effects of the dark radiation ($\sqrt{t}$), the dark matter ($t^{\frac{2}{3}}$) and the dark energy ($e^{\alpha\Lambda t^2}$) at the cosmic scale can be generated by suitable nonlocal gravity models.

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- 1. Cosmological solution for $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$, $k = 0$
  - Scalar curvature is
$$R(t) = \frac{4}{3} t^{-2} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^2 t^2 \qquad (28)$$
  - the Hubble parameter
$$H(t) = \frac{2}{3} t^{-1} + \frac{1}{7}\Lambda t. \qquad (29)$$
  - The eigenvalue problem for operator $\Box$ gives
$$\Box\sqrt{R - 2\Lambda} = -\frac{3}{7}\Lambda\sqrt{R - 2\Lambda} \qquad (30)$$
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$$\ddot{a}(t) = \Big(-\frac{2}{9}t^{-2} + \frac{\Lambda}{3} + \frac{\Lambda^2}{49}t^2\Big)\, a(t). \tag{35}$$

- The expressions (34) implies that $\bar{w}(t) \to -1$ when $t \to \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}t^2}$ describes some effects usually attributed to the dark matter and dark energy.
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- According to above Planck results for t_0 and H_0 in (29) one obtains $\Lambda = 1,05 \cdot 10^{-35}\, s^{-2}$ (in $c = 1$ units).
- The standard formula, $\Lambda = 3\, H_0^2\, \Omega_\Lambda$, gives $\Lambda = 0.98 \cdot 10^{-35} s^{-2}$.
- Friedmann equation and expression (29) for the Hubble parameter give the critical energy density $\rho_c = 8,51 \cdot 10^{-30} g/cm^3$ and the energy density of the dark matter $\bar{\rho} = 2,26 \cdot 10^{-30}\, g/cm^3$.
- It follows that $\bar{\Omega} = \bar{\rho}/\rho_c = 0,265$.
- Ω_v for the visible matter is approximatively $\Omega_v = 0,05$, then $\bar{\Omega}_\Lambda = 1 - \bar{\Omega} - \Omega_v = 0,685$.
- These numbers (procents): $\Omega_v = 0,05$, $\bar{\Omega} = 0,265$, and $\bar{\Omega}_\Lambda = 0,685$, are very close to the 5% of visible matter, 27% of dark matter and 68% of dark energy.
- From expression (29) is possible to find time ($t_m = 21,1 \cdot 10^9$ yr) for which the Hubble parameter has minimum value $H_m = 61,72$ km/s/Mpc.
- Consequently, beginning of the universe expansion acceleration was at $t_a = 7,84 \cdot 10^9$ yr, or in other words at 5.96 billion years ago.

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- Friedmann equation and expression (29) for the Hubble parameter give the critical energy density $\rho_c = 8,51 \cdot 10^{-30} g/cm^3$ and the energy density of the dark matter $\bar{\rho} = 2,26 \cdot 10^{-30}\, g/cm^3$.
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- Scalar curvature is

$$R(t) = 2\Lambda(1 + \frac{2}{3}\Lambda t^2), \tag{36}$$

the Hubble parameter $H(t) = \frac{1}{3}\Lambda t$, $\quad \Box\sqrt{R - 2\Lambda} = -\frac{2}{\sqrt{3}}\Lambda|\Lambda||t|$.

- The eigenvalue problem for operator $\Box$ gives

$$\Box\sqrt{R - 2\Lambda} = -\Lambda\sqrt{R - 2\Lambda} \tag{37}$$

which significantly simplifies analysis of equations of motion.

- From (37) follows

$$\Box^n \sqrt{R - 2\Lambda} = (-\Lambda)^n \sqrt{R - 2\Lambda}, \quad n \geq 0, \tag{38}$$

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$$R_{00} = -\frac{\Lambda^2}{3}t^2 - \Lambda, \quad G_{00} = \frac{\Lambda^2}{3}t^2. \tag{40}$$

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  - Equations of motion holds if and only if

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$$\bar{\rho}(t) = \frac{-\frac{\Lambda}{2} + \frac{3k}{A^2}\, e^{\mp\sqrt{\frac{2}{3}\Lambda}\, t}}{8\pi G}\,, \quad \bar{p}(t) = \frac{\frac{\Lambda}{2} - \frac{k}{A^2}\, e^{\mp\sqrt{\frac{2}{3}\Lambda}\, t}}{8\pi G}. \tag{46}$$

- 2. $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}\, t}\,,\ \Lambda > 0\,,\ k = \pm 1$

- Scalar curvature is

$$R(t) = \frac{6k}{A^2} e^{\mp\sqrt{\frac{2}{3}\Lambda}\, t} + 2\Lambda, \tag{43}$$

the Hubble parameter $H(t) = H \pm \sqrt{\frac{\Lambda}{6}}$ and the following useful equality holds $\Box\sqrt{R - 2\Lambda} = \frac{\Lambda}{3}\sqrt{R - 2\Lambda}$.

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- In this case, we have two solutions:

(1) $a(t) = A\, e^{\sqrt{\frac{\Lambda}{6}}\, t}$ for both $k = +1$ and $k = -1$.

(2) $a(t) = A\, e^{-\sqrt{\frac{\Lambda}{6}}\, t}$, for both $k = +1$ and $k = -1$.

- They are similar to the de Sitter solution $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{3}}\, t}$, $k = 0$, but have time dependent $R(t)$, $\bar{\rho}(t)$ and $\bar{p}(t)$.

- When $t \to +\infty$, parameter $\bar{w}(t) \to -1$ in the case (1) and $\bar{w}(t) \to -1/3$ for solution (2).

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Conclusion.

- These solutions are valid for $\Lambda \neq 0$ and without matter. Some of the solutions are not contained in Einstein's gravity with cosmological constant Λ.
- In particular, solution $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$ deserves further investigation, because it imitates some effects which are usually attributed to the dark matter and the dark energy.
- Calculated cosmological parameters are in good agreement with observations as well. We plan to investigate also other phenomenological aspects according to physical foundations of cosmology.
- In this nonlocal gravity model, analytic function $\mathcal{F}(\Box)$ is rather arbitrary – it is constrained only by a few conditions.
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- We consider perturbations for $k = 0$ with de Sitter solution $a(t) = a_0\, e^{\lambda t}$
- It is usual to introduce the conformal time $d\tau = a(t)dt$, and then $a(\tau) = -\dfrac{1}{\lambda\,\tau}$,

$$\mathrm{d}s^2 = a(\eta)^2\,(-\mathrm{d}\eta^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2).$$

- We take the scalar perturbations of the metric in the form $\hat{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$

$$(h_{\mu\nu}) = a(\eta)^2 \begin{pmatrix} -2\,\phi & -(\nabla B)^T \\ -\nabla B & -2\,\psi \mathrm{Id} + 2\,\mathrm{Hess}\,E \end{pmatrix}$$

  where ϕ, ψ, B and E depend on η, x, y, x.
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- Gauge invariant variables (Bardeen potentials) are given by:

$$\Phi = \phi - \frac{1}{a}(a(B - E'))', \qquad \Psi = \psi + \frac{a'}{a}(B - E'),.$$

- Perturbation of the scalar curvature takes the form

$$\hat{R} = R + \delta R$$

$$\delta R = -R_{\mu\nu} h^{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box)\, h^{\mu\nu}$$

- Perturbations of the equations of motion up to linear order take form

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where $m^2 = M_P^2 + 2f_0(\mathcal{G}'\mathcal{H} + \mathcal{H}'\mathcal{G})$ and
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- To solve the trace equation we use Weierstrass factorization theorem

$$\mathcal{U}(\Box)\,\delta R = \prod_i (\Box - \omega_i^2)\, e^{\gamma(\Box)}\, \delta R = 0,$$

where ω_i^2 are the roots of the equation $\mathcal{U}(\omega^2) = 0$ and $\gamma(\Box)$ is entire function. Moreover, we assume that there is no multiple roots.

- Roots ω_i^2 are obtained as solutions of the eigenvalue problem

$$(\Box - \omega_i^2)\,\delta R = 0.$$

- General solution for δR is the sum over all values of ω_i^2 i.e.,

$$\delta R = \sum_i \delta R_i.$$

- Eigenfunctions take the form

$$\delta R_i = (-k\tau)^{3/2}\left(C_{1i} J_{\nu_i}(-k\tau) + C_{2i} Y_{\nu_i}(-k\tau)\right),$$

where J, Y are Bessel functions of the first and second kind resectively, and $\nu_i = \sqrt{\frac{9}{4} - \frac{\omega_i^2}{\lambda^2}}$.

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$$\delta R_i = (-k\tau)^{3/2}\left(C_{1i} J_{\nu_i}(-k\tau) + C_{2i} Y_{\nu_i}(-k\tau)\right),$$

where J, Y are Bessel functions of the first and second kind resectively, and $\nu_i = \sqrt{\frac{9}{4} - \frac{\omega_i^2}{\lambda^2}}$.

- To solve the trace equation we use Weierstrass factorization theorem

$$\mathcal{U}(\Box)\,\delta R = \prod_i (\Box - \omega_i^2)\, e^{\gamma(\Box)}\,\delta R = 0,$$

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- Bardeen potentials are derived from the following equations

$$-m^2(\Phi-\Psi)+v(\Box)\delta R=0,$$
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- Then Bardeen potentials take the form

$$\Phi+\Psi=\eta(c_1(\cos(\eta)+\eta\sin(\eta))+c_2(-\eta\cos(\eta)+\sin(\eta))),$$
$$\Phi-\Psi=\frac{1}{m^2}\sum_i v(\omega_i^2)\delta R_i, \quad (47)$$

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- For nonlocality $\mathcal{H}(R)=R^p, \mathcal{G}(R)=R^q, 0\neq p,q\in\mathbb{Z}$ we showed that
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- In GTR equations of gravitational waves are derived as perturbations of EOM with respect to Minkowski metric in the form

$$\Box\psi_{\mu\nu} = 0, \qquad \nabla_{\mu}\psi^{\mu\nu} = 0, \tag{48}$$

where $\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}g_{\mu\nu}h$, $h_{\mu\nu} = \delta g_{\mu\nu}$, $h = g^{\mu\nu}h_{\mu\nu}$ and $|h_{\mu\mu}| \ll 1$.

- It was shown that gravitational waves are described in the class of non-local models $\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R)$ with respect to Minkowski metric by the same equations ((48)) as in GTR.

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THANK YOU FOR

YOUR ATTENTION !!!

Non-trivial Christoffel symbols of Friedman – Roberton – Walker metric

$$\Gamma^1_{01} = \frac{\dot a}{a} \qquad \Gamma^2_{02} = \frac{\dot a}{a} \qquad \Gamma^3_{03} = \frac{\dot a}{a}$$

$$\Gamma^0_{11} = \frac{a\,\dot a}{1 - k\,r^2} \qquad \Gamma^1_{11} = \frac{k\,r}{1 - k\,r^2} \qquad \Gamma^2_{12} = \frac{1}{r}$$

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Non-trivial components of curvature tensor

$$R_{0110} = \frac{a\,\ddot{a}}{1 - k\,r^2} \qquad R_{1221} = -\frac{r^2\,a^2\,(\dot{a}^2 + k)}{1 - k\,r^2}$$

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Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & u\,g_{11} & 0 & 0 \\ 0 & 0 & u\,g_{22} & 0 \\ 0 & 0 & 0 & u\,g_{33} \end{pmatrix}, \qquad u = \frac{a\,\ddot{a} + 2\,(\dot{a}^2 + k)}{a^2}$$

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Scalar curvature

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3\,(\dot{a}^2+k)}{a^2} & 0 & 0 & 0 \\ 0 & -v\,g_{11} & 0 & 0 \\ 0 & 0 & -v\,g_{22} & 0 \\ 0 & 0 & 0 & -v\,g_{33} \end{pmatrix}, \qquad v = \frac{2\,a\,\ddot{a} + \dot{a}^2 + k}{a^2}$$

FRW metric EOM EOM-2

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