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Quantisation of free associative dynamical systems

Bi-quantisation of stationary KdV hierarchy and Novikov's equations and non-deformation quantisation of the Volterra sub-hierarchy

Traditional quantisation theories start with classical Hamiltonian systems with variables taking values in commutative algebras and then study their non-commutative deformations, such that the commutators of observables tend to the corresponding Poisson brackets as the (Planck) constant of deformation goes to zero. I am proposing to depart from dynamical systems defined on a free associative algebra $\mathfrak{A}$. In this approach the quantisation problem is reduced to description of two-sided ideals $\mathfrak{J} \subset \mathfrak{A}$ satisfying two conditions: the ideals have to be invariant with respect to the dynamics of the system and to define a complete set of commutation relations in the quotient algebras $\mathfrak{A}_\mathfrak{J} = \mathfrak{A} \diagup \mathfrak{J}$.

To illustrate this approach I'll consider the quantisation problem for N–th Novikov equations and the corresponding finite KdV hierarchy. I will show that stationary KdV equations and Novikov's equations admit two compatible quantisations, i.e. two distinct commutation relations between the variables, such that a linear combination of the corresponding commutators is also a valid quantisation rule leading to the Heisenberg form of quantum equations. The picture is very similar to the bi-Hamiltonian structure in the case of classical integrable equations.

I'll discuss quantisation of the Bogoyavlensky family of integrable N–chains:

$$\frac{du_n}{dt} = \sum_{k=1}^{N} (u_{n+k}\, u_n - u_n\, u_{n-k}), \qquad n \in \mathbb{Z}, \tag{1}$$

quantisation of their symmetries and modifications. In particular, I will show that odd degree symmetries of the Volterra chain ($N = 1$ in (1)) admit two quantisations, one of them corresponds to known quantisation of the Volterra chain, and another one is new and not deformational.

SCIENTIFIC SEMINAR
"DIFFERENTIAL GEOMETRY AND APPLICATIONS"

headed by Academician of RAS Anatoly T. Fomenko

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