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Geodesics on polyhedra: the new results

Abstract: Poincare's conjecture (1905) states that on a smooth convex surface there always exist at least three simple closed geodesics. The hypothesis was proved by Lusternik and Schnirelmann in 1930, and in 1992 Franks and Bangert strengthened this result by establishing the existence of an infinite number of geodesics. The non-smooth case was studied starting with works of Aleksandrov and Pogorelov. In particular, for surfaces of polyhedra, many problems on geodesics admit visual geometric solutions (the works of Post, Galperin, Zalgaller, D. and E. Fuchs, etc.) For nonsmooth surfaces, the Franks-Bangert theorem may fail. For example, a "generic" polytope does not have closed simple geodesics at all. Among such polyhedra is a regular pyramid whose side of the base is not equal to the lateral edge. However, regular polytopes do have simple closed geodesics, all of them (rather, almost all) were listed in the in the work of Dmitry and Ekaterina Fuchs (2007). We give a classification of geodesics and a description of their structure on the surface of an arbitrary simplex. Then we estimate their total number in terms of the flat angles of the faces. In this case, not only simple geodesic are considered, but also the so-called composite geodesics, i.e., families of disjoint simple ones. Next, we will show that polyhedra that have arbitrarily long simple closed geodesics are precisely equilateral tetrahedra. The generalization of this result to arbitrary convex surfaces has been an open problem for more than 10 years, solved in 2018 by Akopyan and Petrunin. In conclusion, we discuss similar problems in spherical space and in Lobachevsky space.

SCIENTIFIC SEMINAR
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