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A rigid body under action of potential and gyroscopic forces: new integrable problems

Equations of motion of a rigid body with a fixed point under action of potential and gyroscopic forces are:

$$J\dot{\boldsymbol{\omega}} = (J\boldsymbol{\omega} + \boldsymbol{\mu}) \times \boldsymbol{\omega} + \boldsymbol{\gamma} \times \frac{\partial V}{\partial \boldsymbol{\gamma}}, \dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma} \times \boldsymbol{\omega},$$

where $\boldsymbol{\omega} = (p, q, r)$ is an angular velocity of a body, J is its inertia tensor with respect to a fixed point, $\boldsymbol{\gamma}$ is a unit vertical vector, $\boldsymbol{\Lambda}$ is a vector of gyroscopic forces, V is a potential of external force field, $\boldsymbol{\mu} = \frac{\partial}{\partial \boldsymbol{\gamma}} \langle \boldsymbol{\Lambda}, \boldsymbol{\gamma} \rangle - \left\langle \frac{\partial}{\partial \boldsymbol{\gamma}}, \boldsymbol{\Lambda} \right\rangle \boldsymbol{\gamma}.$

In the most general case these equations are not integrable, but there are some special cases in which one can obtain integrability.

Except some widely known and successfully solved problems (Euler's top, Lagrange's top, Kowalevsky top, etc) some last works develop new integrable cases. I.e., the book «Rigid body dynamics: A Lagrangian approach with a full survey of integrable problems» by H. M. Yehia recently published by Springer generalizes all new integrable cases developed for last 30 years.

In this talk description of two integrable cases from this book will be given. These cases are obtained as a result of addition a rotor (gyroscopic forces) and applying a potential force to the classical cases of Goriachev–Chaplygin–Sretensky case and Lagranre's top. This fact made the survey much more complicated.

The most important results (topology of isoenergetic manifold, Fomenko invariants, Fomenko–Zieschang invariants, description of singular points of zero rank) are not obtained yet but the direction of survey is rather clear. The conclusion of published results in this area is completed.

SCIENTIFIC SEMINAR "DIFFERENTIAL GEOMETRY AND APPLICATIONS"

headed by Academician of RAS Anatoly T. Fomenko

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