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The relation between the continuity of the lengths of curves and the continuity of distances in the case of boundedly compact metric spaces

This work is devoted to the study of one-parameter deformations of metrics. We assume that the lengths of curves are continuous when the parameter changes, and we study additional conditions that will be sufficient for the continuity of the distances. We start from the presence of the continuity of the lengths of curves, since it is convenient in practice – the continuous dependence of the Riemannian or Finsler metric on the parameter obviously implies the continuity of the lengths of curves, and to obtain the continuity of the distance, it is enough to check the fulfillment of certain conditions. It is shown in the paper that the compactness of space and the continuity of the lengths of curves when changing the parameter is not enough for the continuity of the distances, and an example is given. In addition, we give special conditions, which are sufficient for the continuity of the distances in combination with the boundedly compactness of the space. As an application, we consider Finsler manifolds whose metrics continuously depend on a parameter. We show that sufficient conditions for the continuity of the distance are satisfied on compact Finsler manifolds, from which it follows that the distance function on such manifolds also continuously depends on the parameter. The last result is generalized to complete Finsler manifolds. Since Finsler manifolds are a generalization of Riemannian manifolds, as a corollary we obtain similar results for Riemannian manifolds.