November 2, 2020

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Quaternionic conjugation spaces

There is a considerable amount of examples of spaces X equipped with an involution τ such that the mod 2–cohomology rings $H^{2*}(X)$ and $H^*(X^\tau)$ are isomorphic. Hausmann, Holm, and Puppe have shown that such an isomorphism is a part of a certain structure on equivariant cohomology of X and X^τ, which is called an *H-frame*. The simplest examples are complex Grassmannians and flag manifolds with complex conjugation.

We develop a similar notion of Q-frame which appears in the situation when a space X is equipped with two commuting involutions τ_1, τ_2 and the mod 2-cohomology rings $H^{4*}(X)$ and $H^*(X^{\tau_1,\tau_2})$ are isomorphic. Motivating examples are quaternionic Grassmannians and quaternionic flag manifolds equipped with two complex involutions. We show naturality and uniqueness of Q-framing. We prove that Q-framing can be defined for direct limits, products, etc. of Q-framed spaces. This list of operations contains gluing a disk in $\mathbb{H}^n$ with complex involutions τ_1 and τ_2 to a Q-framed space by an equivariant map of boundary sphere.

An important part of H-frame structure in paper by H.–H.–P. was so called *conjugation equation*. Franz and Puppe calculated the coefficients of the conjugation equation in terms of the Steenrod squares. As a part of a Q-framing we introduce corresponding structure equation and express its coefficients by explicit formula in terms of the Steenrod operations.

SCIENTIFIC SEMINAR
"DIFFERENTIAL GEOMETRY AND APPLICATIONS"

headed by Academician of RAS Anatoly T. Fomenko

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