Geometric description of the Hochschild cohomology of Group Algebras.

A.S. Mishchenko

Moscow State University (M.V. Lomonosov)

A description of the algebra of outer derivations of a group algebra of a finitely presented discrete group is given in terms of the Cayley complex of the groupoid of the adjoint action of the group. This task is a smooth version of Johnson's problem concerning the derivations of a group algebra. It is shown that the algebra of outer derivations is isomorphic to the group of the one-dimensional cohomology with compact supports of the Cayley complex over the field of complex numbers.

On the other hand the group of outer derivation is isomorphic to the one dimensional Hochschild cohomology of the group algebra. Thus the whole Hochschild cohomology group can be described in terms of the cohomology of the classifying space of the groupoid of the adjoint action of the group under the suitable assumption of the finiteness of the supports of cohomology groups.

Spine decompositions and limit results for models with branching structure.

Yanxia Ren

(Peking University)

In this talk, I will describe a 1-spine decomposition and a 2-spine decomposition for the critical Galton-Watson tree and use the 2-spine decomposition to give a probabilistic proof of Yaglom's theorem: conditional on $(Z_n > 0)$, the law of Z_n/n converges to the exponential distribution. Then I will establish a 1-spine decomposition theorem and a 2-spine decomposition theorem for some critical superprocesses. These two kinds of decompositions are unified as a decomposition theorem for size-biased Poisson random measures. These decompositions to can be used to give probabilistic proofs of the asymptotic behavior of the survival probability and Yaglom's exponential limit law for some critical superprocesses. The talk is based on a joint works with Renming Song and Zhenyao Sun.

Statistical Estimation of the Shannon entropy and the Kullback-Leibler divergence.

Alexander Bulinski
(Lomonosov Moscow State University)

Statistical estimation of the Shannon entropy and various divergences are important for applications, e.g., in machine learning, identification of textures inhomogeneities and feature selection. In this regard one can refer, e.g., to the book by V.Bolon-Canedo and A.Alonso-Betanzos (2018), see also a paper by J.R.Vergara and P.A.Estevez (2014). We develop the quite recent works by A.Bulinski, A.Dimitrov (2018, 2019) and A.Bulinski, A.Kozhevin (2018, 2019) to study statistical estimation of the Shannon entropy, mutual information and other divergences. We investigate the asymptotic properties of proposed estimates constructed by means of i.i.d. (vector-valued) observations. For this purpose we apply the techniques involving the nearest neighbor statistics. Special attention is payed to results of computer simulations in the framework of mixed models (see, e.g. F.Coelho, A.P.Braga, M.Verleysen (2016), W.Gao, S.Kannan, P.Viswanath (2018)) comprising the widely used logistic regression.

Mapping classes are almost determined by their finite quotient actions.

Yi Liu
(Peking University)

For any closed surface, we say that two mapping classes are procongruently conjugate if they induce conjugate actions on the outer automorphism group of the profinite completion of the surface group. In this talk, I will sketch a proof of the following result: Every procongruent conjugacy class contains only finitely many conjugacy classes of mapping classes.

On spaces continuously containing topological groups.

Iliadis S.D.

Moscow State University (M.V. Lomonosov)

In the paper [9] it is proved that the group of all self-homeomorphisms of the Hilbert cube with the topology of uniform convergence is universal in the class of all separable metrizable topological groups. Such groups include also the group of isometries of the Urysohn universal metric space (see [10]) and the group of linear isometries of the Gurarij space (see [1]). In [8] it is proved that in the class of all separable metrizable Abelian topological groups there exists a universal element. Moreover, under GCH, it is proved that, for every uncountable cardinal τ, there are universal elements in the class of all metrizable Abelian topological groups of weight $\leq \tau$ and in the class of all topological Abelian groups of weight $\leq \tau$.

However, the problems of the existence of universal elements in the class of all topological groups (see Question 2 of [10]) and in the class of all metrizable topological groups (see Problem 4 of [8]) of a given uncountable weight remain open. Some other problems concerning universal elements in classes of topological groups are given in Section 2.6 of [4]. This fact motivates us to consider, as an alternative of universal topological groups, spaces, which topologically and continuously contain (see below the corresponding definitions) all elements of a given collection $\mathbf{G}$ of topological groups. Such spaces, for the collection $\mathbf{G}$ of all topological groups of the weight $\leq \tau$, were constructed in [4].

Definition. Given an indexed collection $\mathbf{G}$ of topological groups, let Q be a topological space, and let h_Q^G, $G \in \mathbf{G}$, be a topological embedding of G into Q. We say that Q is a *continuosly containing space for* $\mathbf{G}$ *with respect to the collection* $\{h_Q^G : G \in \mathbf{G}\}$ if the following conditions are satisfied: (1) For any points $x, y \in G \in \mathbf{G}$ and each neighbourhood U of $h_Q^G(xy)$ in Q, there exist neighbourhoods V and W in Q of $h_Q^G(x)$ and $h_Q^G(y)$, respectively, such that, for any points $x', y' \in G' \in \mathbf{G}$ such that $h_Q^{G'}(x') \in V$ and $h_Q^{G'}(y') \in W$, we have $h_Q^{G'}(x'y') \in U$; (2) For any $x \in G \in \mathbf{G}$ and each neighbourhood U of $h_Q^G(x^{-1})$ in Q there exists a neighbourhood V of $h_Q^G(x)$ in Q such that, for each $x' \in G' \in \mathbf{G}$ for which $h_Q^{G'}(x') \in V$, we have $h_Q^{G'}((x')^{-1}) \in U$; (3) $\cup\{h_Q^G(G) : G \in \mathbf{G}\} = Q$.

Definition. We say that a topological space Q is *continuously containing for an indexed collection* $\mathbf{G}$ *of topological groups* if, for each $G \in \mathbf{G}$, **there exists** a topological embedding h_Q^G of X into Q such that the space Q is continuously containing space for $\mathbf{G}$ with respect to the collection $\{h_Q^G : G \in \mathbf{G}\}$.

In my talk, we shall concern the following aspects of spaces, continuously containing topological groups.

(1) We construct some *special* spaces, continuously containing for an arbitrary given indexed collection $\mathbf{G}$ of topological groups and give some properties of these spaces which show its relation to other spaces continuously containing for the collection $\mathbf{G}$ (see [5]). The special spaces are obtained in a certain "topological constructive" way and have a "good" structure. The general method of construction of these spaces was given in [2] and widely used in [3].

(2) We define the notion of an *action* of a space, continuously containing topological groups, on a topological space and prove some embedding theorems (see

[6]).

(3) We prove the existence of universal elements in classes of spaces, continuously containing topological groups (see [7]).

References

[1] Itai Ben Yaacov, *The linear isometry group of the Gurarij space is universal*, Proc. Math. Soc. 142(2014), no. 7, 2459-2467.

[2] Stavros Iliadis, *A construction of containing spaces*, Topology and its Applications 107 (2000) 97-116.

[3] S.D. Iliadis, *Universal Spaces and Mappings*, North-Holland Mathematics Studies 198, Elsevier, 2005.

[4] Stavros Iliadis, *On embeddings of topological groups*, Fundamental and Applied Mathematics, Vol 20, No.2, 2015, pp. 105-112 (Russian). Journal of Mathematical Sciences, 2017, 223:6, 720-724 (English).

[5] Stavros Iliadis, *On spaces continuously containing topological groups.* In print.

[6] Stavros Iliadis, *On actions of spaces continuously containing topological groups.* Accepted for publication in Topology and its Applications.

[7] Stavros Iliadis, *Universal elements continuously containing topological groups.* In preparation.

[8] S.A. Shkarin, *On universal Abelian topological groups*, Matematicheskii Sbornik, Vol. 190, No. 7 (1999), 127-144.

[9] V.V. Uspenskij, *A universal topological group with a countable base*, Funktsionanal'nyj analiz i ego prilozhenija (Functional analysis and its applications) 20 (1986), 86-87.

[10] V.V. Uspenskij, *On the group of isometries of the Urysohn universal metric space*, Comment.Math.Univ.Carolinae 31, 1(1990), 181-182.

Fillings of Alexandrov spaces.

Gian Ge

(Peking University)

In this talk, we will talk about geometric properties of Alexandrov spaces that bounds a given positively curved space. Under certain assumptions on the boundary, we get rigidity results. Part of the talk is based on joint works with Ronggang Li.

Symplectic invariants for typical degenerate singularities of integrable systems.

E.A. Kudryavtseva

(Lomonosov Moscow State University)

The talk is based on a joint work with A. Bolsinov and L. Guglielmi (2018). We study parabolic orbits with resonances (also known as Kalashnikov's typical rank-1 singularities) and the singular fibres containing these orbits (known as cuspidal tori) for Lagrangian fibrations on symplectic 4-manifolds. An important property of parabolic orbits is their stability under small integrable perturbations (Kalashnikov 1998, Zung 2000). This is one of the reasons why such singularities can be observed in many examples of integrable Hamiltonian systems. It is well known that, from the smooth point of view, all parabolic orbits of a given resonance k:n are equivalent, i.e. any two parabolic orbits with the same resonance admit fibrewise diffeomorphic neighbourhoods. The same is true for cuspidal tori. We show that, in contrast to non-degenerate singularities (of elliptic, hyperbolic and focus-focus types), there exist parabolic orbits which are locally fibrewise diffeomorphic, but not symplectomorphic. Furthermore, all symplectic invariants of parabolic orbits (with a given resonance) can be expressed in terms of action variables. Finally, we show that the only symplectic semi-local invariant of a cuspidal torus (with a given resonance) is the canonical integer affine structure on the base of the corresponding singular Lagrangian fibration.

Quasi-homomorphisms on mapping class groups.

Jiajun Wang

(Peking University)

We construct infinitely many linearly independent quasi -homomorphisms on the mapping class group of a Riemann surface with genus at least two which vanish on a handlebody subgroup. As a corollary, we disprove a conjecture of Reznikov on bounded generation of Heegaard splittings. Another corollary is that there are infinitely many linearly independent quasi-invariants on the Heegaard splittings of three-manifolds. This is joint work with Jiming Ma.

Interior scattering in graphene.

Ilya Bogaevsky

(Lomonosov Moscow State University)

We consider an isoenergy point source of electrons in graphene with a linear potential and compute the semi-classical asymptotic of their conversion into holes. The Lagrangian submanifold, describing geometric optics of the electrons themselves, has a singularity at the conversion point. Its normal form up to canonical transformations was found by V.I.Arnold in much more general situation.

Partially supported by RFBR and JSPS (research project 19-51-50005).

Entropy degeneracy and measure degeneracy for flows with singularity.

Wenxiang Sun

(Peking University)

While topological entropy is an invariant for equivalent discrete systems, we proved by construction that topological entropy degenerates for flows with singularity:

there exists a pair of equivalent flows so that one has infinite entropy and the other has zero entropy. We characterized entropy degeneracy by measure degeneracy, while the later results to density degeneracy. We constructed a 5-dimensional compact manifold M and a family of vector fields on M which preserve Lebesgue measure and share a unique singularity p. By increasing staying time near p and erducing density of Lebesgue measure outside a neighborhood of p we get Black-Hole Like dynamical systems, which have the following properties: (1) The singularity p attracts infinite density; (2) The event horizon is the manifold M; (3) The time rate of reparametrized time and original time goes to infinity for almost every Lebesgue points; (4) Riemannian metric no longer exists; (5) The black-hole Like dynamics is not observable: any invariant measure except the atomic one supported on the singularity loses its invariance.