Preface ix

Chapter I. PRELIMINARIES 1

1. Singular and Cellular Homology Groups 1

1. Singular chains and homology groups 1
2. Cell complexes and barycentric subdivisions 4
3. Cellular homology and the computation of singular homology of the sphere 9
4. Theorem on the coincidence of singular homology with the cellular homology of a finite cell complex 11
5. Geometric definition of cellular homology groups 13
6. Simplest examples of the computation of cellular homology groups 15

2. Cohomology Groups and Obstructions to Extending Mappings 17

1. Singular cochains and the operator δ 17
2. The problem of the extension of a continuous mapping from a subspace to the whole space 19
3. The obstruction to extending mappings 19
4. Cases of the retraction of a space on a subspace homeomorphic to a sphere 23

3. Fibrations 28

1. Defining a locally trivial fibration 28
2. Examples of fibrations 29
3. The Hopf bundle geometry 30
4. Geometry of the fibration of unit vectors tangent to a sphere and the index of the vector field 34

Chapter II. FUNCTIONS ON MANIFOLDS 41

1. Exact Morse Functions 41

2. Multidimensional Analog of Morse Theory 47

Chapter III. MANIFOLDS OF SMALL DIMENSIONS 55

1. Homeomorphisms of Two-Dimensional Surfaces 55

1. Homeotopy group and classification of three-dimensional manifolds of genus one 55
2. A system of homeotopy group generators of a complete pretzel 61

2. An Algorithm Recognizing the Standard Three-Dimensional Sphere in the Class of Heegard Diagrams of Genus Two 63

1. The Whitehead graphs encoding three-dimensional manifolds. Operations of indices one and two 63
2. Waves separating the vertices and the operation of reducing Whitehead graphs 69
3. The existence of a separating vertex on the Whitehead graphs of genus two, corresponding to the standard three-dimensional sphere. An algorithm recognizing a sphere 72

3. On Solving the Four-Dimensional Poincaré Problem: Any Four-Dimensional Homotopy Sphere is Homeomorphic to the Standard Sphere 74

Chapter IV. MINIMAL SURFACES 93

1. Simplest Properties of Minimal Surfaces 93

1. Plateau physical experiments and methods of obtaining soap films 93
2. Physical principles underlying the formation of soap films 96
3. Extremal properties of soap films and minimality of their area—Properties of the surface of separation between two media 98
4. The surface of separation between two media which are in equilibrium is the surface of constant mean curvature 100
5. Soap films of constant positive curvature and constant zero curvature 103
6. Stable and unstable surfaces 104
7. Plateau experiments on the stability of a liquid column 105
8. Physical realization of a helicoid 109
9. Physical realization of a catenoid and its surgery with changing boundary frame 113
10. The change of the topological type of minimal surfaces depending on their stability or instability 116
11. Two-dimensional minimal surfaces in a three-dimensional space and the first Plateau principle 122
12. Area functional, Dirichlet functional, harmonic mappings, and conformal coordinates 124
13. Singular points of minimal surfaces and three Plateau principles 137
14. Realization of minimal surfaces in nature 144

2. Topological Properties of Minimal Surfaces 146

1. On various aproaches to the concepts of surface and boundary 146
2. Homological boundary of a surface and the role of the coefficient group 148
3. Curious examples of physical stable minimal surfaces, nevertheless retracting on their boundaries 151

3* Realization of the "Bing house" in the form of a minimal surface in R^3 159

4. When does the soap film spanning a boundary frame not contain closed soap bubbles? 166
5. Minimal cones related to singular points of minimal surfaces 169
6. Multidimensional minimal cones 172
7. Minimal surfaces invariant under the action of Lie groups 175
8. Fermat principle, minimal cones, and light rays 179
9. The S. N. Bernshtein problem 187
10. Complex submanifolds as minimal surfaces 189
11. Integral currents 190
12. Spectral bordisms and the multidimensional Plateau problem 193
13. Existence of a minimum in each homotopy class of multivarifolds 197
14. Cases where the Dirichlet problem has no solution for the minimal surface equation of large codimension 198

3. Geometry of Volume Functional and Dirichlet Functional Extremals 203

1. Lower estimate of the volume of minimal surfaces 203
2. Vector field deformation coefficient 206
3. Closed surfaces of non-trivial topological type and least volume 207
4. The cases where there are no local minima of the Dirichlet functional in the class of mappings of homogeneous spaces to an arbitrary Riemannian manifold 210
5. Cases where there are no Dirichlet functional local minima in the class of mappings of an arbitrary Riemannian manifold to a homogeneous space 213

References 216

Index 223